6.2a Integration by Substitution

A change of variables can turn an unfamiliar integral into one that we can evaluate. (The differential matters.)

$$\int f(x)\,dx = \int g(u)\,du$$

$$\int \sin(x)e^{\cos(x)}dx = -e^{\cos x} + c$$

$$\frac{d}{dx}-e^{\cos x} = +\sin x\, e^{\cos x}$$

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$du = -\sin x\, dx$

$\frac{du}{-\sin x} = dx$

$$\int \cancel{\sin x}\, e^{u} \frac{du}{-\cancel{\sin x}}$$

$$\int -e^{u} du = -e^{u} + c$$

$$= -e^{\cos x} + c$$

Nov 30-6:31 PM

$$\int x^2\sqrt{5+2x^3}\,dx = \int \cancel{x^2}\sqrt{u}\,\frac{du}{6\cancel{x^2}} = \frac{1}{6}\int \sqrt{u}\,du$$

$u = 5+2x^3$

$\frac{du}{dx} = 6x^2$

$du = 6x^2\,dx$

$\frac{du}{6x^2} = dx$

$$= \frac{1}{6}\int u^{\frac{1}{2}}\,du$$

$$= \frac{1}{6}\,\frac{u^{3/2}}{3/2} + c$$

$$= \frac{1}{6}\cdot\frac{2}{3}\,u^{3/2} + c$$

$$= \frac{1}{9}\left(5+2x^3\right)^{3/2} + c$$

Nov 30-6:40 PM

$$\int \cot(7x)dx$$

Nov 30-6:46 PM

$$\int \frac{dx}{\cos^2 2x}$$

$$\int \underline{\sec^2 2x}\, dx = \frac{1}{2}\tan 2x + c$$

take der

$$\frac{1}{2}\sec^2 2x \cdot 2$$

Nov 30-6:48 PM

$$\int \cot^2 3x \, dx$$

Nov 30-6:50 PM

$$\int \cos^3 x \, dx$$

Nov 30-6:51 PM

Definite Integrals

$$\int_0^{\frac{\pi}{3}} \tan x \sec^2(x)\,dx = \int_0^{\sqrt{3}} u \cancel{\sec^2 x} \frac{du}{\cancel{\sec^2 x}}$$

$u = \tan x \longrightarrow$ $x=0 \quad u = \tan 0 = 0$

$\frac{du}{dx} = \sec^2 x$ $\quad x = \frac{\pi}{3} \quad u = \tan\frac{\pi}{3} = \sqrt{3}$

$du = \sec^2 x\,dx$

$\frac{du}{\sec^2 x} = dx$

$$= \int_0^{\sqrt{3}} u\,du$$

$$\frac{u^2}{2}\Big|_0^{\sqrt{3}}$$

$$\frac{3}{2} - 0 = \boxed{\frac{3}{2}}$$

alt. soln $\quad \frac{\tan^2 x}{2}\Big|_0^{\pi/3} = \frac{3}{2} - \frac{0}{2} = \frac{3}{2}$

Nov 30-6:52 PM

$$\int_0^1 \frac{x}{x^2-4}\,dx$$

Nov 30-7:00 PM